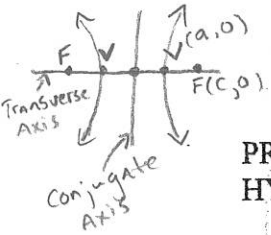
Hyperbola – A collection of ALL points in the plane whose differences of distances from 2 fixed points, called the **foci**, is a constant.



- Transverse axis parallel to x-axis: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
- Transverse axis parallel to y-axis: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Ctr (h,k)

$b^2 = c^2 - a^2$ or $c^2 = a^2 + b^2$

- c = distance from ctr to foci.
- a = distance from ctr to vertices.

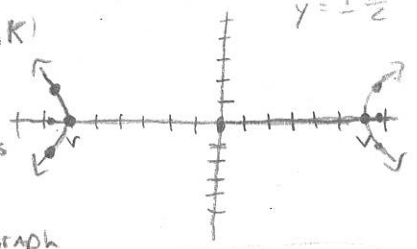
PRE CALC class ex. Sec. 9-4 NAME _____ MOD ____

HYPERBOLAS

1. Graph the hyperbola $\frac{x^2}{36} - \frac{y^2}{9} = 1$. Find its foci. F = $(\pm 3\sqrt{5}, 0)$

- Ctr $(0,0)$
- $a = \sqrt{36} = 6$
- $b = \sqrt{9} = 3$
- $c^2 = a^2 + b^2$
- $c = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$
- Transverse axis is parallel to x-axis b/c x comes first from left to right when looking at the numerators
- Vertices $(h \pm a, k)$ ↳ $(\pm 6, 0)$
- Sub x-coordinates of foci $(\pm 3\sqrt{5})$ in for x to find other pts on the graph
- Foci $(h \pm c, k) = (\pm 3\sqrt{5}, 0)$

$\frac{1}{4} = \frac{y^2}{9}$ $\quad 4y^2 = \frac{9}{4}$ $\quad y = \pm \frac{3}{2}$

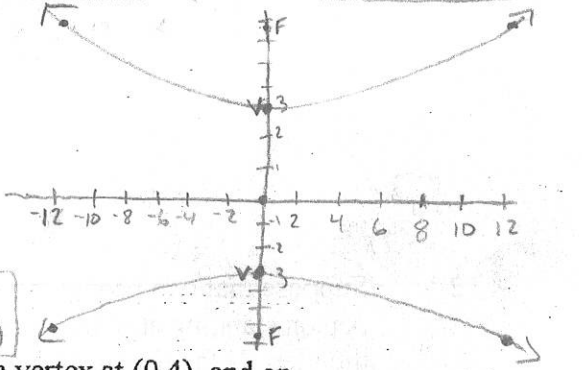


↳ $(3\sqrt{5}, -\frac{3}{2})$; $(-3\sqrt{5}, -\frac{3}{2})$; $(3\sqrt{5}, 3/2)$; $(-3\sqrt{5}, \frac{3}{2})$

2. Graph the hyperbola $\frac{y^2}{9} - \frac{x^2}{36} = 1$. Find its foci. F = $(0, \pm 3\sqrt{5})$

- Transverse axis is parallel to y-axis
- Ctr $(0,0)$
- $a = \sqrt{9} = 3$
- $b = \sqrt{36} = 6$
- $c = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$
- Foci $(h, k \pm c) = (0, \pm 3\sqrt{5})$
- Vertices $(h, k \pm a)$ ↳ $(0, \pm 3)$
- Other pts on graph: sub y-coordinates of Focus in for y + solve for x. $(12, 3\sqrt{5})$; $(-12, 3\sqrt{5})$; $(12, -3\sqrt{5})$; $(-12, -3\sqrt{5})$

$4 = \frac{x^2}{36}$ $\quad 144 = x^2$ $\quad \pm 12 = x$



3. Find the equation of the hyperbola with center at the origin, a vertex at (0,4), and an asymptote with equation $y = \frac{2}{3}x$. $\frac{y^2}{16} - \frac{x^2}{36} = 1$

Ctr $(0,0) = (h,k)$

Vertex $(0,4) = (h, k \pm a)$

Asymptote: $y = \frac{2}{3}x$ ↳ $a = 4$

- Since vertex is on y-axis, use the equation where the transverse axis is parallel to y-axis

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{y^2}{4^2} - \frac{x^2}{6^2} = 1$

- Two oblique asymptotes @ $y = \frac{a}{b}x$ AND $y = -\frac{a}{b}x$
- $y = \frac{4}{b}x$ must equal $y = \frac{2}{3}x$ so b must equal 6.

4. Sketch the hyperbola $x^2 - 9y^2 + 2x + 36y - 44 = 0$. Find the coordinates of its vertices and foci and the equations of its asymptotes.

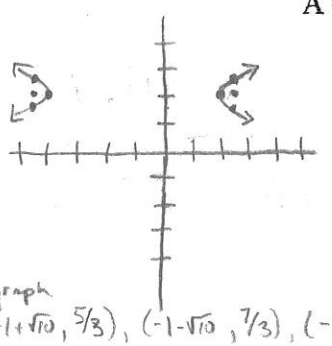
$(x^2 + 2x + 1) - (9y^2 - 36y) = 44$

$(x^2 + 2x + 1) - 9(y^2 - 4y + 4) = 44 + 1 + -9(4)$

$\frac{(x+1)^2}{9} - \frac{9(y-2)^2}{9} = \frac{9}{9}$

$\frac{(x+1)^2}{9} - \frac{(y-2)^2}{1} = 1$

- Ctr $(-1, 2)$
- $a = \sqrt{9} = 3$
- $b = \sqrt{1} = 1$
- $c = \sqrt{9+1} = \sqrt{10}$



- Other pts on graph $(-1+\sqrt{10}, 7/3)$, $(-1+\sqrt{10}, 5/3)$, $(-1-\sqrt{10}, 7/3)$, $(-1-\sqrt{10}, 5/3)$

V = $(2,2) + (-4,2)$

F = $(-1 \pm \sqrt{10}, 2)$

A = $y = \frac{1}{3}x + 2\frac{1}{3}$; $y = -\frac{1}{3}x + 1\frac{2}{3}$

- TV axis parallel to x-axis b/c x comes 1st when looking at numerators from left to right.
- Vertices $(h \pm a, k) \to (-1 \pm 3, 2)$ ↳ $(2,2) + (-4,2)$
- Foci $(h \pm c, k) \to (-1 \pm \sqrt{10}, 2)$
- Asymptotes: $y - k = \pm \frac{b}{a}(x-h)$ $\quad y - 2 = \pm \frac{1}{3}(x+1)$

$\frac{1}{9} = (y-2)^2$ $\quad \pm \frac{1}{3} + 2 = y$